**General Theory of Irrotational Motion**

**Flow:** If  and  be any two points in a fluid, then the of the integral  is called the flow along the path from  to .

When the velocity potential exists i.e. , then we have









.

**Circulation:** The flow round a closed curve is called the circulation round the curve. If is the closed curve, then the circulation is



where  is the velocity vector and the line integral is taken round .

When the velocity potential exists i.e. , then the circulation round the curve is zero.

**Question-01:** State and prove Kelvin’s circulation theorem.

**Answer: Statement:** Circulation along any closed circuit moving in the liquid is constant for all times, provided the external forces are conservative and the density of the liquid is either a function of pressure or a constant.

**Proof:** Let  be any closed circuit moving in the liquid, then



We shall show that



Differentiating (1) with respect to , we have







The equation of motion is



Since the external forces are conservative so we can write



where  is any single valued potential function.

From (2), (3) and (4), we get



Putting  in (5), we get



 



[Since is either a function of pressure of constant]







. (**Proved**)

**Vorticity:** If  is the velocity vector of a fluid particle, then the quantity,  or  is called the vorticity vector or simply vorticity.

**Question-02:** Show that the vorticity is the circulation per unit area.

**OR**

Discuss the relation between circulation and vorticity.

**Answer:** Consider a two dimensional flow in plane. Let  and be the sides of an infinitesimal rectangle of the plane. If the circulation round the rectangular element be , then





Since  is the area of the rectangular element.













So 



But we know, vorticity 

Therefore, 



This is the required relation between circulation and vorticity.

**Permanence of irrotational motion:** When the external forces are conservative and are derivable from a single- valued potential and pressure is a function of density only, then if once the motion of a non-viscous fluid is irrotational, it remains irrotational ever afterwards.

If the motion is irrotational at any instant, the circulation is zero for every closed circuit. But by Kelvin’s circulation theorem the circulation in any closed path moving with the fluid is constant for all times. Then by Stoke’s theorem







The vorticity is zero and therefore the irrotational motion is permanent.

**Problem**

**Problem-01:** In case of two dimensional irrotational motion, the velocity field is given by . What will be the circulation round a square having corners at ,,  and ? Also what will be the circulation round a unit circle with centre at the origin?

**Solution:** The given velocity field is







































Hence the .

**2nd part:** The velocity components are

 and 

The z-component of vorticity is









In the case of two dimensional motion,



 []

.

**Problem-02:** The velocity components of a fluid flow are ,. Calculate the circulation around the closed curve .

**Solution:** The velocity components are

 and 

The z-component of vorticity is







The given curve is





which represents a circle, whose Centre is and radius is .

Therefore the area of the circle is





The circulation is





.

**Problem-03:** The velocity components of a fluid flow are ,. Calculate the circulation around the closed curve .

**Solution:** The velocity components are

 and 

The z-component of vorticity is









The given curve is











which represents an ellipse where and .

Therefore the area of the ellipse is







The circulation is





.

**Problem-04:** If  be the circulation around any closed circuit moving with the fluid, prove that if the external forces have a potential and the pressure is a function of the density alone.

**Solution:** Let  be any closed circuit moving in the liquid, then





Differentiating with respect to , we have







The equation of motion is



Since the external forces have a potential so we can write



From (1), (2) and (3), we get



 



 [Since is a function of density]





; 





. (**Proved**)

**Problem-05:** Find the circulation round a square enclosed by the lines ,  for the flow ,.

**Solution:** The given velocity components are

 and 

















The velocity field is



Also the given lines are

, 

The square enclosed by these lines is drawn in figure.













Hence the 

Where neglecting minus sign because circulation cannot be negative.